Three-loop calculations in the two-dimensional non-linear σ model

1978 J. Phys. A: Math. Gen. 11 1141

(http://iopscience.iop.org/0305-4470/11/6/015)

J. Phys. A: Math. Gen., Vol. 11, No. 6, 1978. Printed in Great Britain

Three-loop calculations in the two-dimensional non-linear σ model

S Hikami and E Brézin
CEN Saclay, BP No 2, 91190 Gif-sur-Yvette, France

Received 30 December 1977

Abstract. The three-loop calculation of the non-linear σ model is performed by the minimal subtraction method near two dimensions. As an application, the third-order expressions for the critical exponents in the $(d-2)$ expansion are obtained.

1. Introduction

The non-linear σ model, with $O(n)$ internal symmetry, is known to be renormalisable and asymptotically free in two dimensions (Polyakov 1975, Migdal 1976). It can be extended through a $(d-2)$ expansion above two dimensions, and for $n>2$ it yields a $(d-2)$ series for the critical exponents corresponding to the second-order phase transition, in which a continuous non-Abelian symmetry is broken.

Brézin and Zinn-Justin (1976a,b this second paper will be referred to as BZ) gave a systematic procedure for the $(d-2)$ expansion and they derived the expressions for the critical exponents up to the second order in the power of $(d-2)$. The procedure of renormalisation for this non-linear σ model was clarified (Brézin *et al* 1976a) and the anomalous dimensions of composite operators were also obtained (Brézin *et al* 1976b).

The calculation for critical dynamics was made by De Dominicis *et al* (1977). The anisotropic case was investigated by Nelson and Pelcovits (1976, 1977).

In this paper, we make a three-loop calculation for this non-linear σ model with $O(n)$ internal symmetry. We calculate the β-function by the method of double power expansion or renormalised temperature t and ϵ $(\epsilon=d-2)$ as in the previous treatment of BZ. In this method, in which the minimal subtraction procedure of 't Hooft (1973) is used, one knows that it is entirely sufficient to determine the coefficients of the renormalisation group equation in two dimensions in order to determine the $(d-2)$ expansion.

In the third-order powers of t, many diagrams appear since the (Euclidean) actions take the following form:

$$A=\int \mathrm{d}x\,\frac{\mu^{d-2}}{2Z_1 t}\left(Z(\partial_\nu\boldsymbol{\pi})(\partial_\nu\boldsymbol{\pi})+\frac{Z^2(\partial_\nu\pi^2)(\partial_\nu\pi^2)}{4(1-Z\pi^2)}-\frac{2HZ_1}{\sqrt{Z}}\sqrt{(1-Z\pi^2)}\right), \tag{1.1}$$

where we eliminated the σ-field by the following relation,

$$\sigma^2+\boldsymbol{\pi}^2=1. \tag{1.2}$$

The π-field has $(n-1)$ components. There are derivative couplings between π-fields in addition to the usual scalar couplings. The last term corresponds to the external magnetic source H; it explicitly breaks the $O(n)$ symmetry and thus gives a non-zero mass to the π-field. However the renormalisation group functions β and ζ are independent of H.

As shown in BZ, the renormalisation group functions β and ζ are written as:

$$\begin{aligned}\beta(t) &= \epsilon t-(n-2)t^2-(n-2)t^3+At^4+O(t^5),\\ \zeta(t) &= (n-1)t+Bt^3+O(t^4).\end{aligned} \tag{1.3}$$

In the large n limit, with nt fixed, it is known that the coefficients of t^3 and t^4 for β, and of t^3 for ζ should vanish. Therefore A and B are at most second-order polynomials in n. Furthermore, in the Abelian case $n=2$, which reduces here to a free field theory, it is known that A and B, as well as all higher coefficients, vanish. Lastly, all the diagrams contributing to the π-field strength renormalisation have at least one closed loop of internal index and thus vanish when n tends to unity. These requirements fix the form of A and B:

$$A=\tau(n-2)(n+\alpha), \qquad B=\lambda(n-2)(n-1). \tag{1.4}$$

The coefficients τ and λ are determined by the knowledge of the $1/n$ expansion. The renormalisation group functions β and ζ are related to ν and η as shown in BZ,

$$1/\nu=-\beta'(t_c), \qquad \eta=-\epsilon+\zeta(t_c). \tag{1.5}$$

The critical temperature, which is obtained from (1.3), becomes

$$t_c=\frac{\epsilon}{n-2}-\left(\frac{\epsilon}{n-2}\right)^2+\left(\frac{\epsilon}{n-2}\right)^3[\tau(n+\alpha)+2]. \tag{1.6}$$

From (1.5) and (1.6), up to order $1/n$, we have

$$\begin{aligned}\frac{1}{\nu} &= \epsilon+\frac{\epsilon^2}{n}-\frac{2\tau}{n}\epsilon^3+\ldots,\\ \eta &= \frac{\epsilon}{n}-\frac{\epsilon^2}{n}+\left(\frac{\lambda}{n}-\frac{1}{4n}\right)\epsilon^3+\ldots.\end{aligned} \tag{1.7}$$

Independent analysis of the $1/n$ expansion (Abe and Hikami 1973, Brézin and Wallace 1973, Ma 1973) gives the coefficients of order ϵ^3/n in (1.7) as $\frac{1}{2}$ for both $1/\nu$ and η. Thus, the constants τ and λ are determined as

$$\tau=-\tfrac{1}{4}, \qquad \lambda=\tfrac{3}{4}. \tag{1.8}$$

In appendix 1, the explicit form of the β-function up to order $1/n$ and to all orders in t is given (Brézin 1977).

Thus, there remains only one unknown constant α in the expression for the β-function. In order to determine α, a three-loop calculation is required†. However, since there remains so little unknown, we can perform the calculation for a special convenient choice of n, and deduce from it the results for any n. It turns out that the unphysical value $n=-1$ is particularly simple, since many diagrams possess a

† A recent (unpublished) calculation of η at order $1/n^2$ has been performed by K Symanzik. It may be used as an independent determination of α. The result agrees and confirms the $n=-1$ procedure that we have used.

combinatorial weight proportional to $(n+1)$. The task is then considerably reduced. The three-loop diagrams, without distinction between derivative couplings and scalar couplings are shown in figure 1. In the case of $n=-1$, the non-vanishing diagrams have only derivative couplings as shown in figure 2.

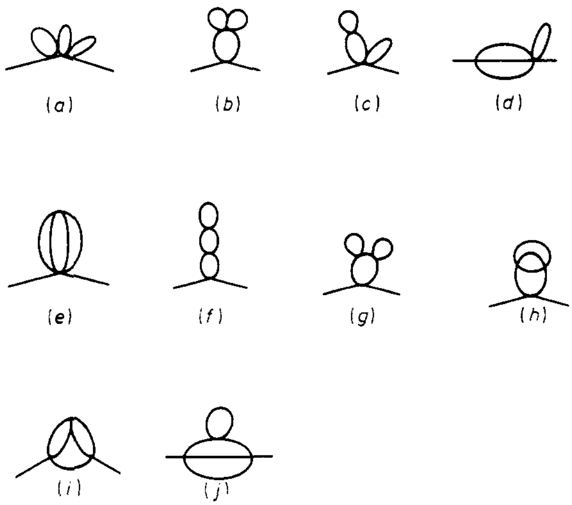

Figure 1. The three-loop diagrams.

By the dimensional regularisation method, we can expand the renormalisation constants Z and Z_1 in a double power series of $1/\epsilon$ and t. The characteristics of the method are that the renormalisation constants only have poles in ϵ for any order of t. Here again the only unknown part is related to the unknown constant α:

$$Z_1=1+\frac{n-2}{\epsilon}t+\left[\left(\frac{n-2}{\epsilon}\right)^2+\frac{n-2}{2\epsilon}\right]t^2+\left[\left(\frac{n-2}{\epsilon}\right)^3+\frac{7}{6}\left(\frac{n-2}{\epsilon}\right)^2 +\frac{(n-2)(n+\alpha)}{12\epsilon}\right]t^3+\mathrm{O}(t^4),$$

$$Z=1+\frac{n-1}{\epsilon}t+\frac{(n-1)(n-\frac{3}{2})}{\epsilon^2}t^2+\left[-\frac{n-1}{3\epsilon^3}\left(-3n^2+\frac{19}{2}n-\frac{15}{2}\right) +\frac{(n-1)(n-2)}{3\epsilon^2}+\frac{(n-2)(n-1)}{4\epsilon}\right]t^3+\mathrm{O}(t^4). \tag{1.9}$$

The values of the coefficients of t^3 in (1.9) are determined by the requirement that β and ζ remain finite in the limit $\epsilon\to 0$ $(d=2)$. Let us recall that β and ζ are related to Z_1 and Z by the following equations:

$$\beta(t)=\frac{\epsilon t}{1+t\frac{\partial}{\partial t}\ln Z_1}, \qquad \zeta(t)=\beta(t)\frac{\partial}{\partial t}\ln Z.$$

This provides a consistency check for three-loop calculations.

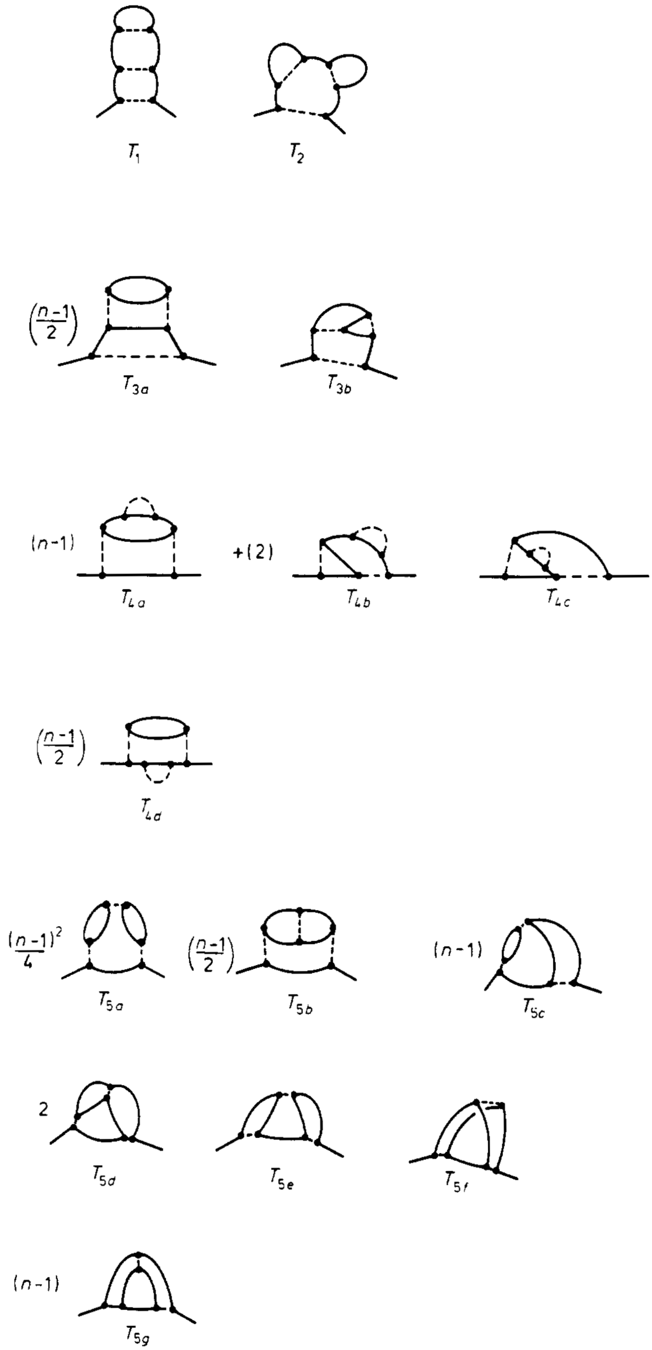

Figure 2. Non-vanishing diagrams for $n = -1$.

2. Calculation

In calculating the diagrams of figure 2, it is convenient to introduce the following quantities,

$$I_l = H^{l-1}\int\frac{1}{(q^2+H)^l}\,d^dq \qquad (l=1,2,3)$$

$$F(v)=\int\frac{1}{(q^2+H)[(q+v)^2+H]}\,d^dq$$

$$G(v)=\int\frac{1}{(q^2+H)^2[(q+v)^2+H]}\,d^dq$$

$$V_l=\int F^2(v)v^l\,d^dv \qquad (l=2,4) \tag{2.1}$$

$$D_l=H^{2-\frac{1}{2}l}\int FGv^l\,d^dv \qquad (l=2,4)$$

$$\rho_j=\int\frac{G(v)}{(v^2+H)^j}\,d^dv$$

$$f_j=\int\frac{F(v)}{(v^2+H)^j}\,d^dv.$$

The calculations of these quantities are given in appendix 2. As in BZ, we abbreviate the geometrical factor $2^{1-d}\pi^{-d/2}/\Gamma(d/2)$ since it can be absorbed into the renormalised temperature t.

By the dimensional regularisation method, we calculate the coefficients of p^2 and H in the diagrams T_i listed below:

$$T_1=(ZZ_1^2t^2)\int\frac{(p+q_1)^2(q_1+q_2)^2(q_2+q_3)^2}{(q_1^2+H)^2(q_2^2+H)^2(q_3^2+H)}\,d^dq_1\,d^dq_2\,d^dq_3$$
$$=(ZZ_1^2t^2)[p^2(I_1^3-6I_1^2I_2+4I_1I_2^2)+H(-5I_1^3+10I_1^2I_2-4I_1I_2^2)] \tag{2.2}$$

$$T_2=(ZZ_1^2t^2)\int\frac{(p+q_1)^2(q_1+q_2)^2(q_1+q_3)^2}{(q_1^2+H)^3(q_2^2+H)(q_3^2+H)}\,d^dq_1\,d^dq_2\,d^dq_3$$
$$=(ZZ_1^2t^2)[p^2(I_1^3-4I_1^2I_2+4I_1^2I_3)+H(-5I_1^3+8I_1^2I_2-4I_1^2I_3)] \tag{2.3}$$

$$T_3=-ZZ_1^2t^2\int\frac{v^2F(v)(p+q_1)^2}{(q_1^2+H)^2[(q_1+v)^2+H]}\,d^dq_1\,d^dv$$
$$+ZZ_1^2t^2\int\frac{v^2(q_1-q_2)^2(p+q_1)^2}{(q_1^2+H)^2(q_2^2+H)[(q_1+v)^2+H][(q_2+v)^2+H]}\,d^dq_1\,d^dq_2\,d^dv$$
$$=(ZZ_1^2t^2)[p^2(I_1^3-3I_1^2I_2+\tfrac{1}{2}V_2-2D_2-\tfrac{3}{2}D_4)$$
$$+H(-5I_1^3+3I_1^2I_2-\tfrac{5}{2}V_2+2D_2+\tfrac{3}{2}D_4)-\tfrac{3}{2}V_4] \tag{2.4}$$

$$T_4/ZZ_1t^2=3p^2I_1^3-7HI_1^3-4p^2I_1^2I_2+6HI_1^2I_2+(2p^2-8H)I_1\int\frac{v^2F}{(p+v)^2+H}\,d^dv$$

$$+4(2H-p^2)I_1H\int\frac{v^2G}{(p+v)^2+H}\,\mathrm{d}^dv+I_1\int\frac{v^2(p\,.\,v)}{(p+v)^2+H}F(v)\,\mathrm{d}^dv$$

$$-2HI_1\int\frac{(p\,.\,v)}{(p+v^2)+H}F(v)\,\mathrm{d}^dv-2HI_1\int\frac{v^2(p\,.\,v)}{(p+v)^2+H}G(v)\,\mathrm{d}^dv$$

$$-4I_1\int\frac{v^4F}{(p+v^2)+H}\,\mathrm{d}^dv+2HI_1\int\frac{v^4F}{[(p+v)^2+H]^2}\,\mathrm{d}^dv$$

$$+6HI_1\int\frac{v^4G}{(p+v)^2+H}\,\mathrm{d}^dv \tag{2.5}$$

$$T_5/ZZ_1t^2=\frac{6}{d-1}p^2I_1^3+\frac{3}{2}\int\frac{v^6F^2}{(p+v)^2+H}\,\mathrm{d}^dv$$

$$+I_1\int\frac{v^2F}{(p+v)^2+H}\,\mathrm{d}^dv\left[\left(4-\frac{2(d+2)}{d(d-1)}\right)p^2-10H\right]$$

$$+6I_1\int\frac{v^2F}{(p+v)^2+H}(p\,.\,v)\,\mathrm{d}^dv-4I_1\int\frac{v^4F}{(p+v)^2+H}\,\mathrm{d}^dv$$

$$+\left[\left(\frac{3}{2(d-1)}-3\right)p^2+5H\right]\int\frac{v^4F^2}{(p+v)^2+H}\,\mathrm{d}^dv-3\int\frac{v^4F^2}{(p+v)^2+H}(p\,.\,v)\mathrm{d}^dv$$

$$+HI_1\int\frac{F}{(p+v)^2+H}\,\mathrm{d}^dv\left[\left(6-\frac{24}{d(d-1)}\right)p^2-6H\right]$$

$$+8HI_1\int\frac{F}{(p+v)^2+H}(p\,.\,v)\,\mathrm{d}^dv$$

$$+H\int\frac{v^2F^2}{(p+v)^2+H}\,\mathrm{d}^dv\left[\left(\frac{2(d+5)}{d(d-1)}-7\right)p^2+\frac{11}{2}H\right)$$

$$-7H\int\frac{v^2F^2}{(p+v)^2+H}(p\,.\,v)\,\mathrm{d}^dv$$

$$+\left[\left(\frac{24}{d(d-1)}-4\right)p^2+2H\right]H^2\int\frac{F^2}{(p+v)^2+H}\,\mathrm{d}^dv$$

$$-4H^2\int\frac{F^2}{(p+v)^2+H}(p\,.\,v)\,\mathrm{d}^dv \tag{2.6}$$

Following the dimensional regularisation method, we are concerned only with the divergent terms up to order $1/\epsilon$. The constant terms in (2.2)–(2.6) are not needed. The total summation of (2.2)–(2.6), denoted as T becomes

$$T/ZZ_1t^2=\left[\left(20-\frac{46}{d}+\frac{6}{d-1}-\frac{2(d+2)}{d(d-1)}\right)p^2-8H\right]I_1^3+\left[\left(\frac{28}{d}-27\right)p^2+17H\right]I_1^2I_2$$

$$+\left[\left(\frac{12}{d}+\frac{3}{2(d-1)}-4\right)p^2+H\right]V_2+(-2p^2+2H)D_2+\left(-\frac{3}{2}p^2+\frac{3}{2}H\right)D_4$$

$$+(4p^2-4H)(I_1^2I_3+I_1I_2^2)+\left(-\frac{1}{2}+15Hf_1\right)p^2I_1. \tag{2.7}$$

These three-loop terms are expanded in powers of $1/\epsilon$ (the calculation is given in appendix 2); it yields

$$T=p^2t^2\left[-\frac{5}{3\epsilon^3}-\frac{17}{2\epsilon^2}+\frac{1}{\epsilon}\left(g-\frac{15}{4}-\frac{5}{24}\pi^2\right)\right]+Ht^2\left[\frac{14}{3\epsilon^3}+\frac{6}{\epsilon^2}+\frac{1}{\epsilon}\left(\frac{17}{2}\pi^2+\frac{3}{2}\right)\right] \tag{2.8}$$

with $g=\frac{1}{2}-15Hf_1$.

The renormalisation constants Z and Z_1 in (1.9) are determined in order that the two-point vertex function $\Gamma^{(2)}(p, H)$ remains finite when ϵ goes to zero. For $n=-1$, the two-point vertex function $\Gamma^{(2)}(p, H)$ is calculated up to two-loop order:

$$\begin{aligned}\Gamma^{(2)}=&\frac{Z}{Z_1t}\left(p^2+\frac{HZ_1}{\sqrt{Z}}\right)+(p^2Z-HZ_1\sqrt{Z})\left(\frac{Z_1H}{\sqrt{Z}}\right)^{d/2-1}I_1\\&-(ZZ_1t)\left\{\left[\left(4-\frac{6}{d}\right)p^2-\frac{2Z_1H}{\sqrt{Z}}\right]I_1^2+\left(-2p^2+\frac{2Z_1H}{\sqrt{Z}}\right)I_1I_2\right.\\&\left.+\left(\frac{10}{d}-2\right)Hf_1p^2\left(\frac{Z_1H}{\sqrt{Z}}\right)^{d-2}\right\}+\mathrm{O}\ (\text{three-loop}).\end{aligned} \tag{2.9}$$

Using the expansions of Z and Z_1 in (1.9) up to order t^2, and the calculations in appendix 2, we obtain the terms of order t^2 in (2.9) as

$$\begin{aligned}\Gamma_2^{(2)}(p,H)=&p^2t^2\left(\frac{5}{3\epsilon^3}+\frac{17}{2\epsilon^2}+\frac{3}{\epsilon}+\frac{\alpha-1}{4\epsilon}+\frac{5\pi^2}{24\epsilon}+\frac{15}{\epsilon}Hf_1\right)\\&+Ht^2\left[-\frac{14}{3\epsilon^3}-\frac{6}{\epsilon^2}-\left(\frac{7}{12}\pi^2+\frac{3}{2}\right)\frac{1}{\epsilon}\right].\end{aligned} \tag{2.10}$$

The constant α in (2.10) is determined by the following equation:

$$T+\Gamma_2^{(2)}(p,H)=0. \tag{2.11}$$

Thus, we obtain

$$\alpha=2. \tag{2.12}$$

3. Summary and discussion

With the argument in §1, the renormalisation group functions $\beta(t)$ and $\zeta(t)$ are obtained as

$$\begin{aligned}&\beta(t)=\epsilon t-(n-2)t^2-(n-2)t^3-\tfrac{1}{4}(n-2)(n+2)t^4+\mathrm{O}(t^5),\\&\zeta(t)=(n-1)t+\tfrac{3}{4}(n-1)(n-2)t^3+\mathrm{O}(t^4).\end{aligned} \tag{3.1}$$

The critical point t_c is given by $\beta(t_c)=0$, $\beta'(t_c)<0$, thus

$$t_c=\frac{d-2}{n-2}-\left(\frac{d-2}{n-2}\right)^2+\frac{6-n}{4}\left(\frac{d-2}{n-2}\right)^3+\mathrm{O}(d-2)^4. \tag{3.2}$$

The critical exponents ν and η are next determined through the relations (1.5), and the expressions for ν and η are

$$\frac{1}{\nu}=(d-2)+\frac{(d-2)^2}{(n-2)}+\frac{(d-2)^3}{2(n-2)}+O(\epsilon^4) \tag{3.3}$$

$$\eta=\frac{d-2}{n-2}-\frac{n-1}{(n-2)^2}(d-2)^2+\frac{n(n-1)}{2(n-2)^3}(d-2)^3+O(\epsilon^4). \tag{3.4}$$

Using the scaling law relation

$$2\beta=\nu(d-2+\eta) \tag{3.5}$$

we obtain

$$\beta=\frac{n-1}{2(n-2)}-\frac{(n-1)(d-2)}{(n-2)^2}+\frac{3}{2}\frac{(n-1)}{(n-2)^3}(d-2)^2+O(\epsilon^3). \tag{3.6}$$

The direct use of these expansions in three dimensions is difficult, since the series seems very divergent. However, as in the other ϵ-expansion near four dimensions one could try some Padé–Borel extrapolation. For applying these expansions in the case $n=3$ and $d=3$, we perform the Borel transformation:

$$\frac{1}{\nu}=\int_0^\infty e^{-t}\left(\epsilon t+\frac{(\epsilon t)^2}{2}+\frac{(\epsilon t)^3}{12}\right)dt \tag{3.7}$$

and with an appropriate Padé approximant ([1, 1] Padé involves a pole in the positive axis), we obtain at least the right order of magnitude:

$$\frac{1}{\nu}=\int_0^\infty e^{-t}\frac{t}{1-\frac{1}{2}t+\frac{1}{6}t^2}dt\simeq 1\cdot 25. \tag{3.8}$$

The exponent η is also analysed as

$$\eta=\int_0^\infty e^{-t}t\left(\frac{1-\frac{1}{2}t}{1+\frac{1}{2}t}\right)dt\simeq 0\cdot 11. \tag{3.9}$$

However the lack of sign oscillations in the coefficients of the ϵ-expansion of $1/\nu$ may be a signal that this ϵ expansion is not Borel summable and that the physics may only be recovered with the addition of exp $(-c/\epsilon)$ terms (Brézin 1977).

Acknowledgments

One of us (SH) wishes to thank Nishina Memorial Foundation for a research fellowship and would like to acknowledge the hospitality of the Theoretical Physics Division at Saclay.

Appendix 1

The β-function is written as in the large n limit,

$$\beta(t)=\epsilon t-(n-2)t^2-t^2g(nt). \tag{A.1}$$

The critical point t_c is given by

$$t_c = \frac{\epsilon}{n} + \frac{2\epsilon}{n^2} - \frac{\epsilon}{n^2} g(\epsilon) + O\left(\frac{1}{n^3}\right). \tag{A.2}$$

From (1.5), (A.1) and (A.2), we obtain:

$$\frac{1}{\nu} = \epsilon + \frac{\epsilon^2}{n} g'(\epsilon) + O\left(\frac{1}{n^2}\right). \tag{A.3}$$

Explicit expression for ν in $1/n$ expansion is given by

$$\frac{1}{\nu} = \epsilon + \frac{1}{n} 4\epsilon^2 \left(\frac{1+\epsilon}{2+\epsilon}\right) \frac{\sin\frac{1}{2}\epsilon\pi}{\pi\epsilon} \frac{\Gamma(1+\epsilon)}{[\Gamma(1+\frac{1}{2}\epsilon)]^2} + O\left(\frac{1}{n^2}\right). \tag{A.4}$$

Thus, the expression for $\beta(t)$ turns out to be

$$\beta(t) = \epsilon t - (n-2)t^2 - t^2 \int_0^{nt} 4\left(\frac{1+x}{2+x}\right) \frac{\sin\frac{1}{2}x\pi}{\pi x} \frac{\Gamma(1+x)}{[\Gamma(1+\frac{1}{2}x)]^2} \, dx$$

$$\simeq \epsilon t - (n-2)t^2 - nt^3 - \frac{n^2}{4} t^4 + \frac{n^3}{12} t^5 + \frac{2\zeta(3) - 1}{32} n^4 t^6 + \dots. \tag{A.5}$$

Appendix 2

The quantities I_l, V_l and D_l in (2.1) are expanded in powers of $1/\epsilon$ as

$$I_1 = -\frac{H^{\epsilon/2}}{\epsilon}\left(1 + \frac{\pi^2}{24}\epsilon^2\right)$$

$$I_2 = \frac{H^{\epsilon/2}}{2}\left(1 + \frac{\pi^2}{24}\epsilon^2\right)$$

$$I_3 = \frac{H^{\epsilon/2}}{4}\left(1 - \frac{\epsilon}{2} + \frac{\pi^2}{24}\epsilon^2\right)$$

$$V_2 = H^{3\epsilon/2}\left(-\frac{4}{3\epsilon^3} - \frac{\pi^2}{6\epsilon}\right)$$

$$V_4 = H^{1+3\epsilon/2}\left[\frac{16}{3\epsilon^3} - \frac{4}{3\epsilon^2} + \left(2 + \frac{2}{3}\pi^2\right)\frac{1}{\epsilon}\right]$$

$$D_2 = \left(-\frac{3}{8}\epsilon\right) V_2$$

$$D_4 = -\frac{1}{4H}\left(1 + \frac{3}{2}\epsilon\right) V_4.$$

The constant terms ρ_j and f_j in two dimensions are given by

$$Hf_1 = -\tfrac{1}{2}\int_0^1 \frac{\ln x}{1 - x + x^2} \, dx = 0{\cdot}58598$$

$$H^2 f_2 = \tfrac{1}{3} H f_1$$

$$H^3 f_3 = -\tfrac{1}{12} + \tfrac{1}{3} H f_1$$

$$H^4 f_4 = -\tfrac{2}{27} + \tfrac{7}{27} H f_1$$

$$H^2 \rho_1 = \tfrac{1}{3} H f_1$$

$$H^3 \rho_2 = \tfrac{1}{12}$$

$$H^4 \rho_3 = -\tfrac{1}{72} + \tfrac{1}{9} H f_1$$

References

Abe R and Hikami S 1973 *Prog. Theor. Phys.* **49** 442
Brézin E 1977 *Budapest Conference on Particle Physics, July 1977*
Brézin E and Wallace D J 1973 *Phys. Rev.* B **7** 1967
Brézin E and Zinn-Justin J 1976a *Phys. Rev. Lett.* **36** 691
—— 1976*b* *Phys. Rev.* B **14** 3110
Brézin E, Zinn-Justin J and Le Guillou J C 1976a *Phys. Rev.* D **14** 2615
—— 1976b *Phys. Rev.* B **14** 4976
De Dominicis C, Ma S K and Peliti L 1977 *Phys. Rev.* B **15** 4313
't Hooft G 1973 *Nucl. Phys.* B **61** 455
Ma S K 1973 *Phys. Rev.* A **7** 2172
Migdal A A 1976 *Sov. Phys.-JETP* **42** 743
Nelson D R and Pelcovits R A 1976 *Phys. Lett.* **57A** 23
—— 1977 *Phys. Rev.* B **16** 2191
Polyakov A M 1975 *Phys. Lett.* **59B** 79